

Tipping the balances of a small world

Luís M. A. Bettencourt

*Center for Theoretical Physics, Massachusetts Institute of Technology
77 Massachusetts Avenue, Cambridge MA 02139, USA*

Recent progress in the large scale mapping of social networks is opening new quantitative windows into the structure of human societies. These networks are largely the result of how we access and utilize information. Here I show that a universal decision mechanism, where we base our choices on the actions of others, can explain much of their structure. Such collective social arrangements emerge from successful strategies to handle information flow at the individual level. They include the formation of closely-knit communities and the emergence of well-connected individuals. The latter can command the following of others while only exercising ordinary judgement.

In recent years there has been growing interest in the quantitative structure of human societies. It has emerged that we are part of heterogeneous networks or graphs (1-4), sets of links that connect each one of us to all our acquaintances. Not all people are alike: some live almost isolated, most belong to distinguishable communities (1,5) and a small fraction of the population is made up of exceptionally well connected individuals (6). Social networks have the remarkable property that one can reach anyone else through a very small number of connections – the famous six degrees of separation (7,8). Watts and Strogatz (1) coined the term small world networks, to describe the class of graphs with these properties that describe human societies.

These findings beg important questions: Why are social networks invariably clustered in communities? Why are there individuals with such different connectivity? Answering these puzzles requires tying the morphology of social networks to their function (2,9,10). Similar problems occur in the study of other complex networks, for example, dealing with gene and protein-protein interactions (11-13), metabolism (14,15), ecosystems (16,17) (foodwebs) and neural activity. Thus understanding the simultaneous robustness and adaptability of these complex systems in the light of their function is a general problem at the forefront of the current scientific agenda across many disciplines (9).

The difficulty of this approach consists in defining the function of each of these complex networks in a way that captures their essence and simultaneously permits quantitative progress. Clearly many details of social behaviour, in particular, appear too rich and our understanding of them remains too qualitative to fall in this class. There are however important well documented exceptions.

A familiar situation is having to choose between seemingly equivalent options, at least given the amount of information and time at our disposal (18). In practice many of our decisions fall in this class. This leads to a degeneracy of choice, typical also of situations when relevant information is difficult to discriminate from too much noise, or when it cannot be trusted. In these situations we often rely on the observation of the actions of others we know as the basis for our decisions (19-23). This strategy has two important advantages: we can be sure not to do worse than most of the people we know and, in addition, we may actually join a winning trend early and profit from it.

Recently this type of discriminating imitation has become the focus of an extensive empirical literature in economy (19) and the social sciences. Bikhchandani, Hirshleifer and Welch (20,21) collected a vast amount of empirical evidence that establishes the

universal importance of the choices of others in influencing our own and were able to model this phenomenon in very simple terms. They dubbed the formation of the trend or fad that often results an *information cascade*; a process whereby sequential individual choices propagate a piece of information through the entire population (22). This phenomenon is also often likened (qualitatively) to the spread of an epidemic (10,22). Interestingly, information cascades lead to the spontaneous formation of large consensus where there are *a priori* no individual preferences.

Here I use an implementation of these ideas (23) consisting of a population of N agents, facing a choice among L labels. At each time step individuals compare the relative growth rate of their label to that of one of their immediate acquaintances', chosen at random. If the latter's growth rate (the trend's relative momentum) is greater the agent switches to his neighbor's trend; otherwise he keeps his. The model has one additional ingredient: if a trend slows down individuals may decide to take a risk in something new (an empty label). This effect is modeled by p_{crit} , the relative growth rate below which non-conformism sets in. Here $p_{crit}=10^{-5}$, which results in population wide trends or cascades (23). References to other related dynamical implementations (22) (including models of *herding*) and additional discussion are given elsewhere (23).

Typical dynamics (23) are characterized by cycles alternating population disorder, when many different trends coexist, and order, when most of the population falls into the same label. Both collective states of order and disorder are dynamically unstable making the evolution very sensitive to chance events (20,21,23). As a result it becomes very difficult in practice for an external observer to profit from the reckoning that agents are following trends, especially when the number of choices becomes large.

To explore the effects of the underlying geometry on the dynamics I generate (binary) artificial social networks as *small world graphs* (1,24). These are random graphs with clustering: N individuals are represented as nodes, each with an average number of connections z . Clustering is produced by dividing the population into communities, each characterized by an average higher degree of internal connections per node z_{in} than external z_{out} connections ($z=z_{in}+z_{out}$).

In addition to measure correlations between parts of the population it is useful to define a label state vector

$$\vec{v}_c = (N_1^c, N_2^c, \dots, N_L^c) / a_c, \quad a_c = \sqrt{\sum_{i=1}^L (N_i^c)^2},$$

where N_i^c denotes the number of individuals in label i belonging to group c . The natural inner product

$$\langle \vec{v}_c | \vec{v}_{c'} \rangle = (a_c a_{c'})^{-1} \sum_{i=1}^L (N_i^c N_i^{c'})$$

is a (positive definite) measure of the correlation between different groups, see Fig. 1.

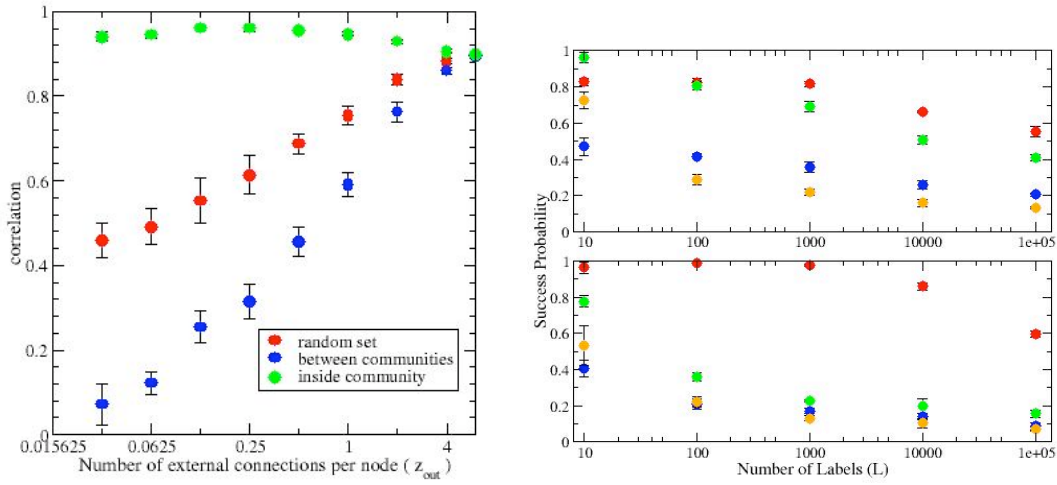


Fig. 1 The correlation between two halves of the same community (green), halves of distinct communities (blue) and a set of individuals with random connections (red) for $N=256$, $L=1000$, $z=8$, divided in 4 communities (see text). The correlation inside a community is always close to 1. The correlation and synchronization of choices between distinct communities is low for small z_{out} , becoming higher as the number of connections between them increases. Individuals with random connections display intermediate correlation. For high z_{out} the original communities merge together. Error bars denote standard deviations over a set of 20 network realizations and many cascade cycles. Fig. 2 The success rate of several criteria for predicting the next winning trend at the time when the dominant movement decays. The next winning trend is not easily determined as the largest secondary trend (blue), the fastest (orange) or even the one with the largest product of size and momentum (green). The best predictor is the choice of the hub (red), particularly as the number of choices L becomes large. The upper panel refers to lower hub visibility (his input is considered on average by each individual with probability $p_{hub}=1/8$, each time), the lower panel to higher visibility ($p_{hub}=1/2$). Error bars are as in Fig. 1.

Fig. 1 shows the correlation between several subsets of the population, within a community, between two distinct communities and for a control set of individuals with random connections. It is now clear why it is a good defensive strategy to belong to a tightly knit collective: communities are *islands of information coherence*. Thanks to the large redundancy of personal connections inside the community the coherence of local information is preserved and personal deviations inside the group are small compared to those to the outside. This remains true even if a few individuals or connections are lost.

Comforting as it may be to keep up with our neighbours it may actually be better to be a step ahead. As we discussed above this is a tall order, even if one is fully informed of the state of the whole population. Figure 2 shows the success rate of several criteria to predict the emerging new trend at the particularly important time when a former dominant movement collapses. All criteria based on the full knowledge of the state of population at this particular time (the largest secondary trend, the fastest growing trend or the trend with the largest product of momentum and size) are far from good and become very poor for large number of competing choices L .

Interestingly there is a simple alternative solution – it relies on *connections*, not reasoning or information. I examine this scenario by introducing a new well-connected

individual into the population, a network hub, as in Fig 3. The hub bases his decisions, like any other agent, on the state of an average number of other individuals z , but his choices can be seen by everybody else. What is particular about the evolution is illustrated in Fig. 2 and 4: the hub is exceptionally good at picking the next winning trend early, before it becomes dominant – the perfect winning strategy.

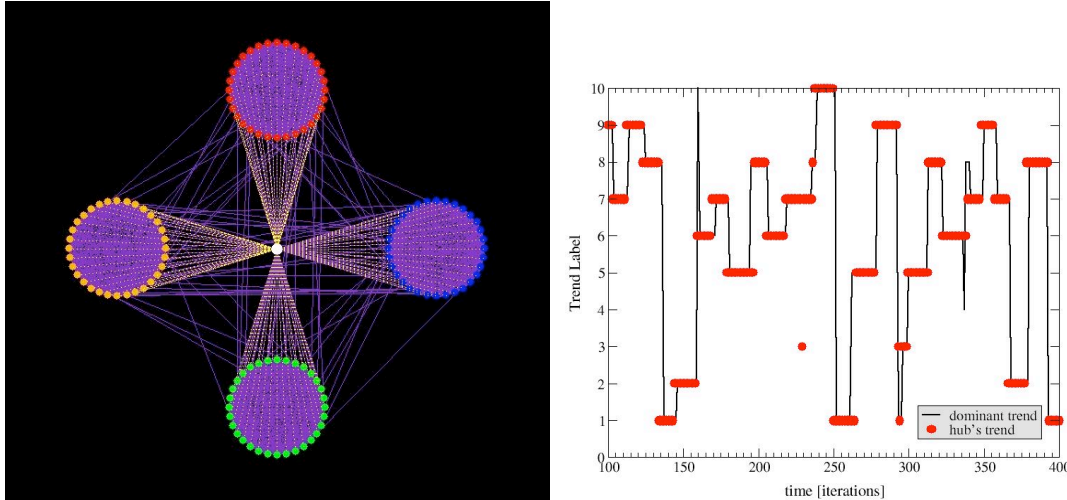


Fig. 3 An example of a (binary) social network with $N=128$, $z=4$, divided into 4 communities (red, blue, green, orange) and with a hub (central node). Here the state of the hub is seen by all individuals with probability $p_{\text{hub}}=0.25$ each time, but has input from $z=4$ individuals. As such the actions of the hub are very visible but not better informed. Fig. 4 Evolution of the dominant trend (solid) and the hub's trend (red) for $L=10$. The hub invariably picks the next dominant trend correctly and early, qualities that reinforce his social position.

However the hub is, by construction, neither better informed nor animated by superior decision making. This apparent paradox is easily dispelled: the hub's actions are very visible to others. Any reasonable decision on his part (the adoption of any growing label) has a large probability of being followed by many ($\sim Np_{\text{hub}}$) and thus to *make* the winning trend. This property is independent of the underlying community structure and is enhanced for larger populations (larger N), as long as $p_{\text{hub}}(N)$ such that $d(Np_{\text{hub}})/dN > 0$. Thus, it is popularity, not knowledge or reasoning, that leads to the most successful strategy in an environment where many courses of action are equally viable.

Given some memory the hub's successes reinforce his position and (apparent) foresight. Each correct 'prediction' encourages others to heed his choices and follow. This reinforces the hub's popularity, allowing him to pick the next winning trend with greater certainty and so on: the process is self-reinforcing. It also naturally leads to a specific form of preferential attachment (25), where the most connected node – the best trend predictor – is preferred. Thus, under choice degeneracy, one should expect the appearance of well-connected, very visible individuals as a social network evolves.

Nodes with an exceptionally large connectivity are a common property of other complex networks, including scale-free graphs (26) describing e.g. WWW, power-grids and the large scale features of protein-protein interactions *in vivo*. It has been pointed (27,28) out that such networks' utmost fragility is due to the loss of these key nodes. Trend dynamics shows how this fragility may only be apparent in social networks. The

hub is a common node, only its degree of outgoing connections is exceptionally large. I argued above that there is a fundamental instability for a common individual to be promoted to this position. Because social connections are rearranged on much faster time scales than nodes (29), upon loss of a hub a new one can quickly develop from another node and the structural integrity of the network will be preserved after a short transient. Moreover the addition of a second well-connected node dealing with the same information reduces the predictability of emerging trends unless the two hubs work in tandem (as would happen under specific assortative mixing (29)) and so forth. It is however perfectly natural for separate hubs to coexist if they relate to different social dimensions (30), i.e. if they deal with different types of information. In this way the large scale structure of human societies, when averaged over time and social dimensions may be characterized by many hubs with varying reaches and interdependencies. These properties may lead to the emergence of interesting scale-invariances in large social networks associated with decision making and information flow.

Observing the actions of others is a universal simple mechanism that allows us to handle imperfect information in our complex social environment to make difficult decisions. We can protect ourselves from the tyranny of fashions by associating into tightly knit communities or we may try to set trends by influencing the choices of others through our social connections. Here I showed that these successful individual strategies lead to stable social arrangements, which coincide with some of the most notable observed structures of social networks. Trend dynamics breaks the degeneracy of our individual choices and leads to the spontaneous formation of collective movements. Whenever concerted social action is more productive than the sum of individual efforts social hubs may become the social mechanism that facilitates the creation of consensus most promptly and predictably.

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